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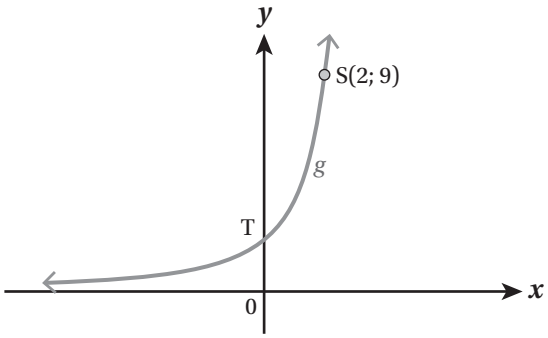
Contents

Functions (Exponential & logarithmic)

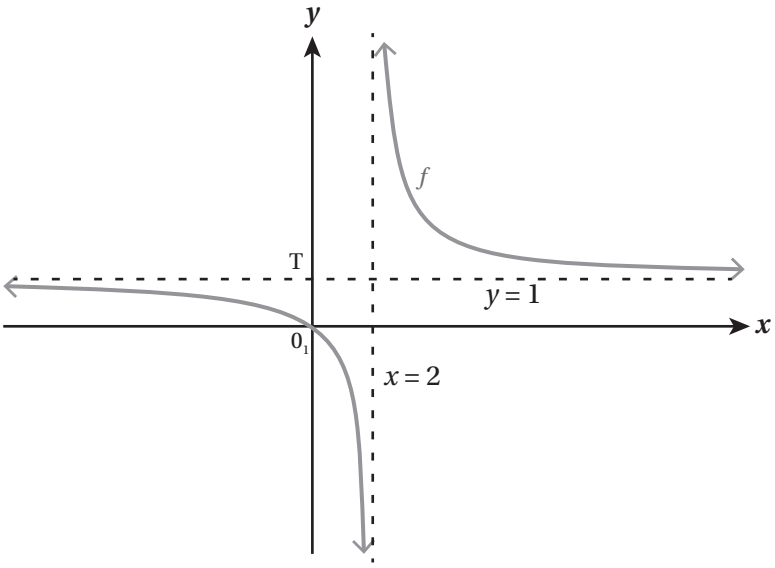
TOPIC 1

Exam-type questions

The graph of $g(x) = a^x$ is drawn in the sketch. The point S(2; 9) lies on g. T is the y-intercept of g.



1. Write the coordinates of T.
2. Calculate the value of a.
3. The graph h is obtained by reflecting g in the y-axis. Write the equation of h.
4. Write the values of x for which $0 < \log x < 1$.
5. The function f, defined by $f(x) = \frac{a}{x+p} + q$, has the following properties:
   The range of f is $y \in \mathbb{R}, y \neq 1$.
   The graph f passes through the origin.
   P$(\sqrt{2} + 2; \sqrt{2} + 1)$
   - **5.1** Write down the value of q.
   - **5.2** Calculate the values of a and p.
   - **5.3** Sketch a neat graph of this function. Your graph must include the asymptotes, if any.



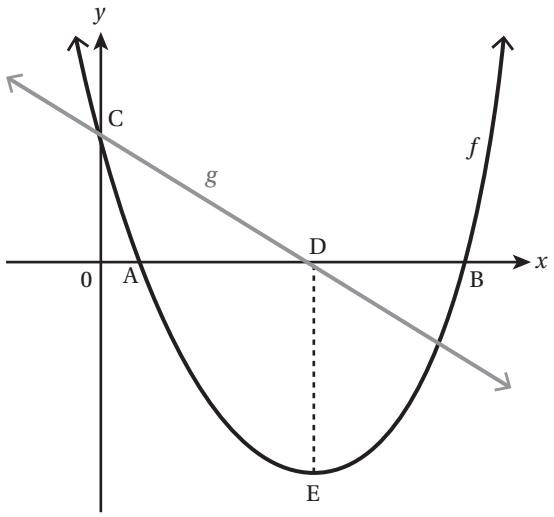
Functions including polynomials

TOPIC 2

Exam-type questions

1. Below are the graphs of $f(x) = (x - 4)^2 - 9$ and a straight line g.

- A and B are the x-intercepts of f, and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.



1.1 Write down the coordinates of E.

1.2 Calculate the coordinates of A.

1.3 M is the reflection of C in the axis of symmetry of f. Write the coordinates of M.

1.4 Determine the equation of g in the form $y = mx + c$.

1.5 Write down the equation of g^{-1} in the form $y = \ldots$

1.6 For which values of x will $x(f(x)) \leq 0$?

Number patterns, sequences and series

TOPIC 3

Exam-type questions

1. Find the sum of all multiples of 3 between 1 and 300.

2. The first three terms of an arithmetic sequence are -1; 2 and 5.

2.1 Determine the nth term, T_n, of the sequence.

2.2 Calculate T_{35}.

2.3 Evaluate $\sum_{k=1}^{12} T_k$.

3. Given the geometric sequence: 30; 10; $\frac{10}{3}$; ...

3.1 Determine n if the nth term of the sequence is equal to $\frac{10}{729}$.

3.2 Calculate S_∞ for the series $30 + 10 + \frac{10}{3} + \ldots$

4. Consider the sequence $x - 2$; 20; $5x$; ... Calculate the value of x $(x > 0)$ if:

4.1 the sequence is arithmetic

4.2 the sequence is geometric.

5. **5.1** What is the least value of n for which $\sum_{k=1}^{n} 8(2)^{k-1} > 30$?

5.2 What is the greatest value of n for which $\sum_{k=1}^{n} 8(2)^{k-1} < 400$?

6. A quiz has 100 questions.

6.1 Jake answers questions numbered $5k - 2$ where $k \in \mathbb{N}$. How many questions does he answer?

6.2 Trent does questions 7; 11; 15; 19; ... 99. Find a formula to generate this sequence and then determine how many questions Trent answers.

7. Given the sequence 4; 4; 10; 12; 16; 36; ...

7.1 Write down the next two terms of the sequence.

7.2 Calculate $T_{28} - T_{27}$.

8. 2, m and n are the first three terms of an arithmetic sequence. If each term is reduced by 1, the resulting sequence is geometric. Find the two sequences.

9. The common difference of an arithmetic sequence is 4.
Prove that $\sqrt{S_{2n} - 2S_n} = 2n$.

10. Calculate $\sum_{k=1}^{3} 4^{k-1} + \sum_{k=1}^{10} (k-1) + \sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^{k-1}$

Finance, growth and decay

Exam-type questions

1. Donald invests R5 000 in a savings account for 2 years. The investment earns 10% p.a. compounded quarterly.

1.1 What is the quarterly interest rate for his investment?

1.2 Calculate the amount in Donald's savings account at the end of 2 years (to the nearest rand).

2. A company bought a new printing machine at the beginning of 2015 for R24 000. It depreciates by 11,5% on a straight line basis.

2.1 Calculate the book value of the machine at the beginning of 2019.

2.2 In how many years would the machine have a book value of zero (to the nearest year)?

2.3 If depreciation was calculated on a reducing balance with the same rate, calculate how many years it would take to reach the same book value (calculated in 2.1).

3. Mr Salie buys a car for R255 000. It depreciates in value on a reducing balance by 12,5% p.a.

3.1 Calculate the value of the car after 3 years (to the nearest rand).

3.2 After how many years will Mr Salie's car be worth R100 000?

4. Reece buys a car costing R192 000 and takes out a 5-year loan with interest charged at 12% p.a. compounded monthly.

4.1 Calculate Reece's monthly instalments.

4.2 After he has paid 45 instalments, he decides to settle the balance on the car loan. Calculate the lump sum he needs to pay after he has paid his 45th instalment.

5. A company buys a vehicle for R450 000.

5.1 Calculate the annual rate of depreciation on a reducing balance if the vehicle is valued at R243 736,90 after 4 years.

5.2 If the average rate of the price increase is 8,1% p.a., calculate the expected cost of a new vehicle in 4 years' time.

5.3 The vehicle that cost R450 000 needs to be replaced in 4 years' time. The old vehicle will be traded in and a sinking fund is set up to cover the replacement cost of the vehicle. The first payment will be made at the end of the 13th month and the last payment will be made at the end of the 48th month. The sinking fund earns interest at 6.2% p.a. compounded monthly.
Calculate the monthly payment to the fund.

Differential calculus

TOPIC 5

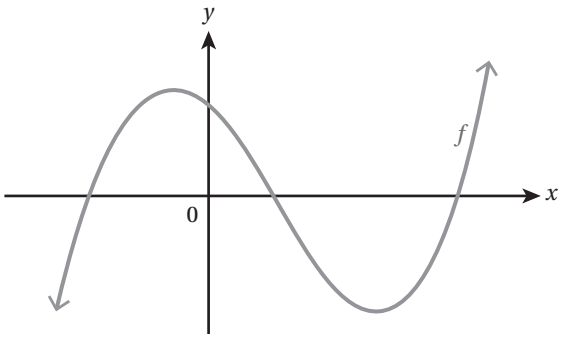
Exam-type questions

1. **1.1** Determine $f'(x)$ from first principles if $f(x) = 4x^2$.

1.2 Determine:

1.2.1 $D_x\left[\frac{x^2 - 2x - 3}{x + 1}\right]$ **1.2.2** $f''(x)$ if $f(x) = \sqrt{x}$

2. The sketch below represents the curve of $f(x) = x^3 + bx^2 + cx + d$. The solutions of the equation $f(x) = 0$ are -2; 1 and 4.



2.1 Calculate the values of b, c and d.

2.2 Calculate the x-coordinate of B, the maximum turning point of f.

2.3 Determine an equation for the tangent to the graph of f at $x = -1$.

2.4 Sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch.

2.5 For which value(s) of x is $f(x)$ concave upwards?

3. Given: $f(x) = -3x^3 + x$.
Calculate the value of q for which $f(x) + q$ will have a maximum value of $\frac{8}{9}$.

Probability

TOPIC 6

Exam-type questions

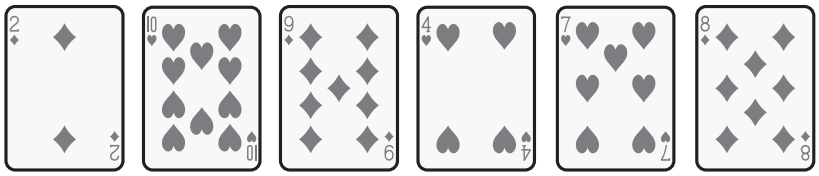
1. Veli and Bongi are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

1.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day.

1.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day.

1.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will your response be to him? Show ALL calculations.

2. The cards below are placed from left to right in a row.



2.1 In how many different ways can these 6 cards be randomly arranged in a row?

2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions?

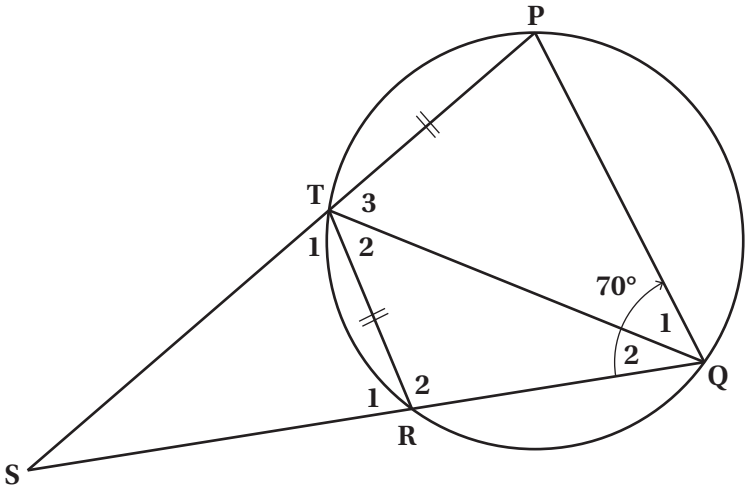
2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another.

Euclidean geometry and measurement

TOPIC 7

Exam-type questions

1. In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. SQP = 70°



1.1 Calculate, with reasons, the size of:

1.1.1 $\hat{T}_1$

1.1.2 $\hat{Q}_1$

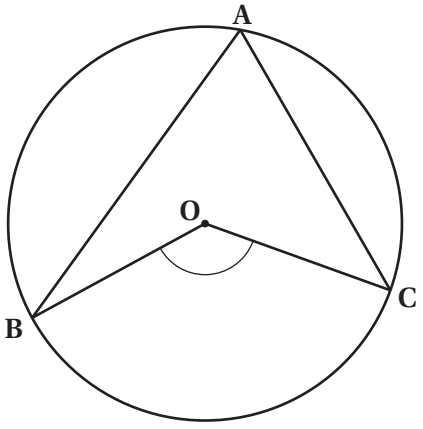
1.2 If it is further given that PQ || TR:

1.2.1 Calculate, with reasons, the size of $\hat{T}_2$.

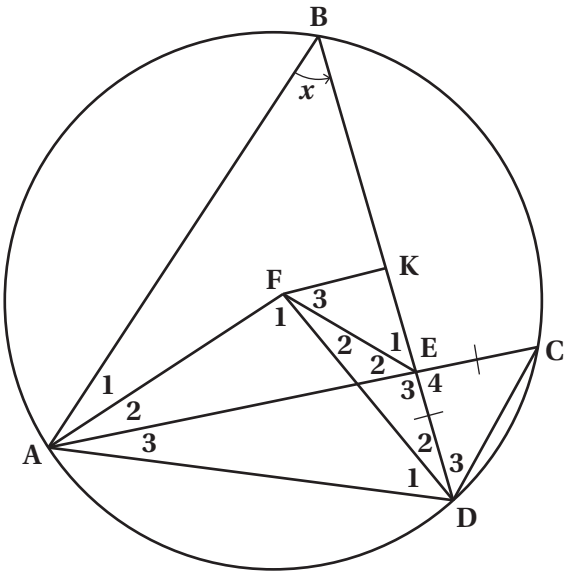
1.2.2 Prove that $\frac{TR}{TS} = \frac{RQ}{RS}$.

2. In the diagram, 0 is the centre of the circle with A, Band C drawn on the circle.

2.1 Prove the theorem which states that $B\hat{O}C = 2\hat{A}$.



2.2 In the diagram, the circle with centre F is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint A of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let $\hat{B} = x$.



2.2.1 Determine, with reasons, the size of EACH of the following m terms of x:

(**a**) $\hat{F}_1$

(**b**) $\hat{C}$

2.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral.

2.2.3 Prove, with reasons, that $\hat{F}_3 = x$.

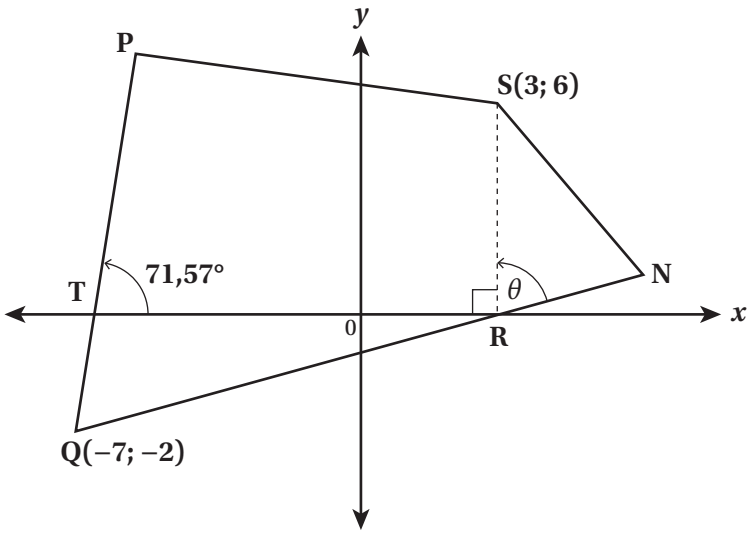
2.2.4 If area $\triangle AEB = 6{,}25 \times$ area $\triangle DEC$, calculate $\frac{AE}{ED}$.

Analytical geometry

TOPIC 8

Exam-type questions

1. In the diagram, P, Q(−7; −2), R and S(3; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71{,}57°$ and $S\hat{R}N = \theta$.



Determine:

1.1 The equation of SR

1.2 The gradient of QP to the nearest integer

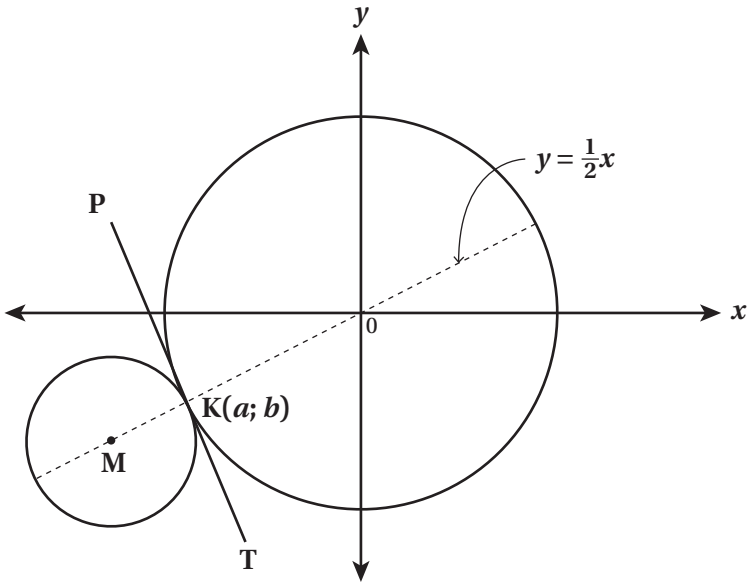
1.3 The equation of QP in the form $y = mx + c$

1.4 The length of QR. Leave your answer in surd form.

1.5 $\tan(90° - \theta)$

1.6 The area of $\triangle$RSN, without using a calculator.

2. In the diagram, PKT is a common tangent to both circles at K(a; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



2.1 Write down the length of OK in surd form.

2.2 Show that K is the point $(-12; -6)$.

2.3 Determine:

2.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$

2.3.2 The coordinate of M

2.3.3 The equation of the smaller circle in the form $(x - a)^2 + (y - b)^2 = r^2$

2.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points?

2.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M$(-16; -8)$.

Trigonometry

TOPIC 9

Exam-type questions

Simplify the following without the use of a calculator:

1. If $\tan 41° = t$, write down in terms of t the value of:
1.1 $\tan 221°$ **1.2** $\sin 41°$ **1.3** $\cos 82°$.

2. **2.1** Complete the formula $\cos(A + B) =$ .....
2.2 Deduce a formula for $\cos 2A$ in terms of $\cos A$.
2.3 Show that $\cos 3A = 4\cos^3 A - 3\cos A$.
2.4 Hence, find, in terms of k, the value of $\cos 51°$, given that $1 - \sin^2 17 = k$.

3. If $13\sin A - 5 = 0$ and $\tan B = -\frac{3}{4}$ where $A \in [90°; 270°]$ and $B \in [90°; 270°]$, determine, without the use of a calculator, the values of the following.
3.1 $\cos A$
3.2 $\cos(A + B)$

4. **4.1** Prove: $\frac{\sin x + \sin 2x}{\cos x + \cos 2x + \cos 0°} = \tan x$
4.2 Prove that $\tan A + \frac{\cos A}{\sin A} = \frac{2}{\sin 2A}$.

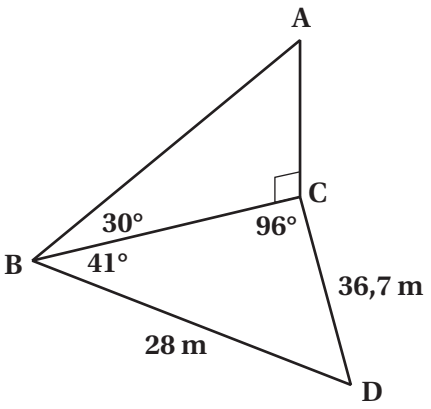
5. **5.1** Prove that $\sin(45° + x) \cdot \sin(45° - x) = \frac{1}{2}\cos 2x$.
5.2 Hence determine the maximum value of $\sin(45° + x) \cdot \sin(45° - x)$.

6. **6.1** Prove that $\cos(A - 30°) - \cos(A + 30°) = \sin A$.
6.2 Hence, determine the general solution to the equation:
$\cos(A - 30°) - \cos(A + 30°) = 2\sin^2 A$.

7. Show that the equation $\sin(x + 60°) + 2\cos x = 0$ can be written as $\tan x = -4 - \sqrt{3}$.

8. **8.1** Show that $\cos(x + 30°) = 2\sin x$ can be written as $\sqrt{3}\cos x = 5\sin x$.
8.2 Hence, solve $\cos(x + 30°) = 2\sin x$ for $x \in [0°; 360°]$, giving your answers rounded off to one decimal place.
8.3 On the same set of axes, draw sketch graphs of $y = \cos(x + 30°)$ and $y = 2\sin x$ for $x \in [0°; 360°]$.
8.4 Use your graphs to determine for which values of x, $2\cos(x + 30°) \cdot \sin x < 0$.

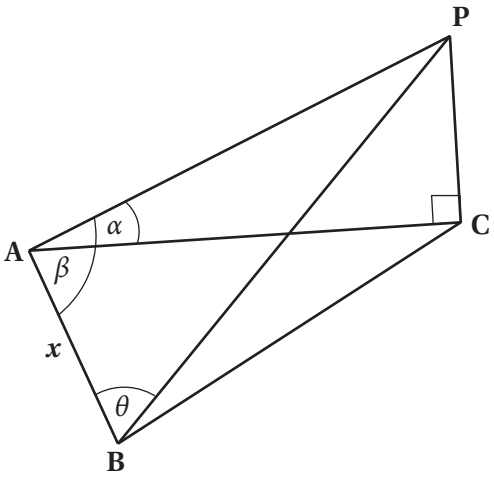
9. B and D are points in the same horizontal plane as C, the foot of a flagpole AC. The angle of elevation of A from B is 30°. $D\hat{C}B = 96°$, $C\hat{B}D = 41°$ and BD = 28 m.
Calculate the length AB.



10. Two ropes lead from the top of a vertical tower PC to points A and B on the ground.
AB = x, $P\hat{A}C = \alpha$, $P\hat{A}B = \beta$ and $P\hat{B}A = \theta$.

10.1 Prove PC = $\frac{x \cdot \sin\alpha \cdot \sin\theta}{\sin(\beta + \theta)}$.

10.2 If $\alpha = 42°$, $\beta = 75°$, $\theta = 68°$ and $x = 7$ m, calculate the height of the tower.



Trigonometry

TOPIC 10

Exam-type questions

1. An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students at college X is shown in the table below.

Days	1	2	3	4	5	6	7	8	9	10
Units of blood	45	59	65	73	79	82	91	99	101	106

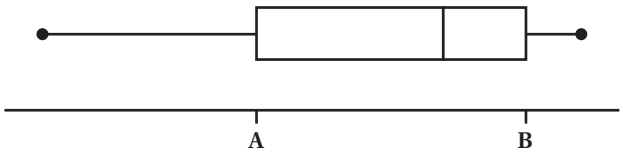
1.1 Calculate:

1.1.1 The means of the units of blood donated per day over the period of 10 days

1.1.2 The standard deviation of the data

1.2 How many days is the number of units of blood donated at college X outside one standard deviation from the mean?

1.3 The number of units of blood donated by the students of college X Is represented in the box-and-whisker diagram below.



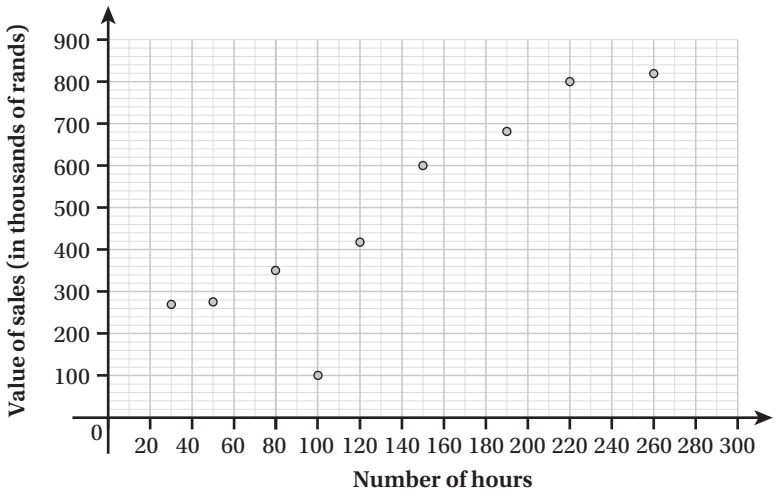
1.3.1 Describe the skewness of the data.

1.3.2 Write down the values of A and B, the lower quartile and the upper quartile respectively, of the dataset.

1.4 It was discovered that there was an error in counting the number of units of blood donate by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the 10 days?

2. The table below shows the number of hours that the sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

Number of hours	30	50	80	100	120	150	190	220	260
Value of sales	270	275	376	100	420	602	684	800	820



2.1 Identify an outlier in the data.

2.2 Calculate the equation of the least squares regression line of the data.

2.3 The sales representative forgot to record the sales of one of his clients. Predict the value of this client's sales (in thousands of rands) if he spent 240 hours with him during the year.

2.4 What is the expected increase in sales for **each** additional hour spent with a client?

Answers

The complete worked solutions to the questions are available on the USB device containing the digital component or can be downloaded from our website: freeresources.oxford.co.za (go to Answer Guides).

Topic 1 Functions

Exam-type questions

1. $a^0 = 1$ T(0; 1)

2. $g(x) = a^x$

$9 = a^2$

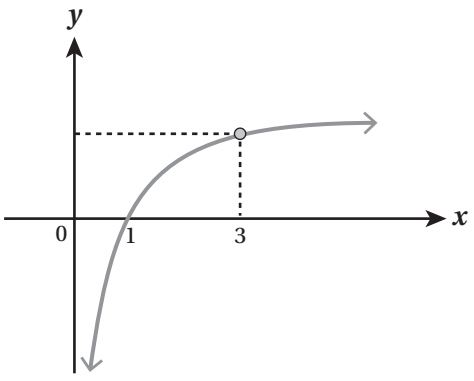
$a = 3 \quad a > 0$

3. $y = \left(\frac{1}{3}\right)^x$ or $y = 3^{-x}$

4. $3^0 < 3^{\log_3 x} < 3^1$

$1 < x < 3$

OR



$1 < x < 3$

5.1 $q = 1$

5.2 Subs (0; 0) $0 = \frac{a}{0+p} + 1$

$\frac{a}{p} = -1$

$a = -p$

Subs P:

$$\sqrt{2} + 1 = \frac{a}{\sqrt{2} + 2 + p} + 1$$

$$\sqrt{2} = \frac{a}{\sqrt{2} + 2 + p} + 1$$

$$2 + 2\sqrt{2} + \sqrt{2}p = a$$

$$2 + 2\sqrt{2} = a - p\sqrt{2} = a + a\sqrt{2}$$

$$2(1 + \sqrt{2}) = a(1 + \sqrt{2})$$

$\therefore a = 2; p = -2$

Topic 2 Functions including polynomials

Exam-type questions

1.1 E(4; −9)

1.2

$f(x) = (x-4)^2 - 9$

$(x-4)^2 - 9 = 0$

$(x-4)^2 = 9$

$(x-4) = \pm 3$

$x = 7$ or $x = 1$

A(1; 0)

OR

$f(x) = (x-4)^2 - 9$

$0 = x^2 - 8x + 16 - 9$

$0 = x^2 - 8x + 7$

$(x-7)(x-1) = 0$

$x = 7$ or $x = 1$

A(1; 0)

1.3 C(0; 7)

M(8; 7)

1.4 C(0; 7)

D(4; 0)

$m = \frac{7-0}{0-4}$ or $m = \frac{0-7}{4-0}$ or $0 = 4m + 7$

$m = -\frac{7}{4}$ $m = -\frac{7}{4}$ $m = -\frac{7}{4}$

$y - 0 = -\frac{7}{4}(x-4)$

$y = -\frac{7}{4}x + 7$

1.5

$g: y = -\frac{7}{4}x + 7$

$g^{-1}: x = -\frac{7}{4}y + 7$

$7y = -4x + 28$

$y = -\frac{4}{7}x + 4$

OR

g^{-1} is the straight line through (0; 4) and (7; 0).

$y = mx + 4$

$0 = 7m + 4$

$y = -\frac{4}{7}x + 4$

1.6 $x \cdot f(x) \leq 0$

$\therefore x \leq 0$ or $1 \leq x \leq 7$

Topic 3 Number patterns, sequences and series

Exam–type questions

1. $n = 99$ and $S_{99} = 14\,850$

2.1 $T_n = 3n - 4$

2.2 $T_{35} = 101$

2.3 186

3.1 $n = 8$

3.2 $S_\infty = 45$

4.1 $x = 7$

4.2 $x = 10$

5.1 $n = 3$

5.2 $n = 5$

6.1 20 questions

6.2 24 questions

7.1 22; 108

7.2 $T_{28} = T_{14}$ of GP $= 4(3)^{13} = 6\,377\,292$
$T_{27} = T_{14}$ of AP $= 6(14) - 2 = 82$
$T_{28} - T_{27} = 6\,377\,210$

8. AP = 2; 2; 2; ...
GP = 1; 1; 1; ...

9. $d = 4$

$$S_n = \frac{n}{2}(2a + (n-1)(4))$$
$$= 2n^2 - 2n + an$$

and $$S_{2n} = \frac{2n}{2}(2a + (2n-1)(4))$$
$$= 8n^2 - 4n + 2an$$

$S_{2n} - S_n = 4n^2$

$\therefore \sqrt{S_{2n} - 2S_n} = 2n$

10. $21 + 45 + \frac{4}{3} = 66\frac{4}{3}$

Topic 4 Finance, growth and decay

Exam–type questions

1.1 2,5%

1.2 R6 092

2.1 R12 960

2.2 9 years

2.3 5 years

3.1 R170 830

3.2 7 years

4.1 R4 270,93

4.2 R59 217

5.1 14,21%

5.2 R614 490,66

5.3 F = R370 753,76
monthly payment = R9 397,11

Topic 5 Differential calculus

Exam-type questions

1.1

$$f(x+h) = 4(x+h)^2$$
$$f(x+h) - f(x) = 4(x+h)^2 - 4x^2$$
$$= 4(x^2 + 2xh + h^2) - 4x^2$$
$$= 4x^2 + 8xh + 4h^2 - 4x^2$$
$$= 8xh + 4h^2$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\left[\frac{8xh + 4h^2}{h}\right]$$
$$= \lim_{h\to 0}\left[\frac{h(8x+4h)}{h}\right]$$
$$= 8x$$

OR

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{\text{h}}$$
$$= \lim_{h\to 0}\left[\frac{4(x+h)^2 - 4x^2}{h}\right]$$
$$= \lim_{h\to 0}\left[\frac{4x^2xh + 4h^2 - 4x^2}{h}\right]$$
$$= \lim_{h\to 0}\left[\frac{h(8x+4h)}{h}\right]$$
$$= 8x$$

1.2.1

$$\text{D}_x\frac{x^2-2x-3}{x+1}$$
$$= \text{D}_x\frac{(x-3)(x+1)}{x+1}$$
$$= \text{D}_x(x-3)$$
$$= 1$$

1.2.2

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$
$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$
$$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$$

2.1

$$f(x) = (x+2)(x-1)(x-4)$$
$$= (x^2 + x - 2)(x-4)$$
$$= x^3 + x^2 - 2x - 4x^2 - 4x + 8$$
$$= x^3 - 3x^2 - 6x + 8$$
$$b = -3;\ c = -6;\ d = 8$$

2.2

$$f(x) = x^3 - 3x^2 - 6x + 8$$
$$f'(x) = 0$$
$$3x^2 - 6x - 6 = 0$$
$$x^2 - 2x - 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= 2 \pm \frac{\sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{12}}{2}$$
$$x = -0{,}73$$

2.3

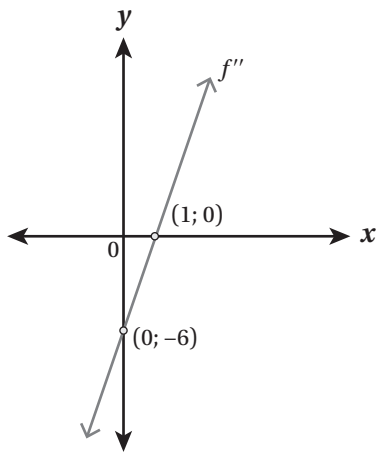
$$f(x) = x^3 - 3x^2 - 6x + 8$$
$$f(-1) = (-1)^3 - 3(-1)^2 - 6(-1) + 8$$
$$= 10$$
$$f'(-1) = 3(-1) - 6(-1) - 6$$
$$= 3$$
$$y - 10 = 3(x+1)$$
$$y = 3x + 13$$

OR

$$f(-1) = (1)(-2)(-5)$$
$$= 10$$

2.4

$$f''(x) = 6x - 6$$



2.5

f concave upwards
$$f''(x) > 0$$
$$6x - 6 > 0$$
$$x > 1$$

3. $f(x) = -3x^3 + x$

$-9x^2 + 1 = 0$

$x = \frac{1}{3}$ or $x = -\frac{1}{3}$

Maximum of f will beat $x = \frac{1}{3}$.

$f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)$

$= \frac{2}{9}$

Maximum of $f(x) + q$ will also be at $x = \frac{1}{3}$.

$f\left(\frac{1}{3}\right) + q = \frac{8}{9}$

$\frac{2}{9} + q = \frac{8}{9}$

$q = \frac{6}{9}$

$= \frac{2}{3}$

For $f(x) + q$ to have a maximum of $\frac{8}{9}$ the value of q has to be $\frac{2}{3}$.

Topic 6 Probability

Exam-type questions

1.1 Let the event Veli arrive late for school be V.
Let the event Bongi arrive late for school be B.
$P(V \text{ or } B) = 1 - 0{,}7$
$= 0{,}3$

1.2 $P(V \text{ or } B) = P(V) + P(B) - P(V \text{ and } B)$
$0{,}3 = 0{,}25 + P(B) - 0{,}15$
$P(B) = 0{,}2$

1.3 $P(V) \times P(B) = 0{,}25 \times 0{,}2$
$= 0{,}05$
$P(V) \times P(B) \neq P(V \text{ and } B)$
V an B are NOT independent.

2.1 $6! = 720$

2.2 Number of arrangements
$= 3! \times 3! \times 2$
$= 72$

2.3 P(hearts next to each other)
$= \frac{3! \times 4!}{6!}$
$= \frac{144}{720}$
$= \frac{1}{5}$ or 0,2 or 20%

OR P(hearts next to each other)
$= \frac{4! \times 3! \times 3!}{6!}$
$= \frac{144}{720}$
$= \frac{1}{5}$ or 0,2 or 20%

Topic 7 Euclidean geometry and measurement

Exam-type questions

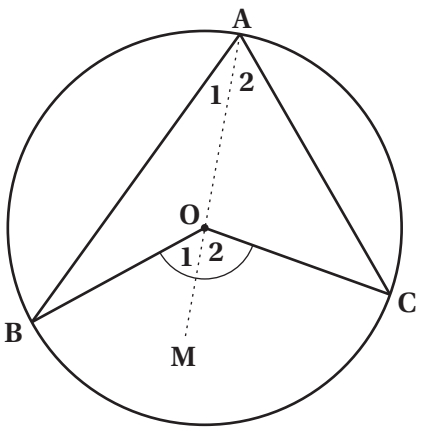
1.1.1 $\hat{T}_1 = 70°$ (ext. $\angle$ of cyclic quad)

1.2.1 $\hat{T}_2 = \hat{Q}_1 = 35°$ (alt. $\angle$s)

1.1.2 $\hat{Q}_1 = \hat{Q}_2 = 35°$ (equal chords, equal $\angle$s)

1.2.2 $\frac{PT}{TS} = \frac{QR}{RS}$ (prop theorem)
$\therefore \frac{TR}{TS} = \frac{QR}{RS}$ (PT = TR)

2.



2.1 Construction:
AO is drawn and produced to M.

$\hat{O}_1 = \hat{A}_1 + \hat{B}$ (ext. $\angle$ of $\triangle$)

But $\hat{A}_1 = \hat{B}$ ($\angle$s opp = radii)

$\therefore \hat{O}_1 = 2\hat{A}_1$

Similarly: $\hat{O}_2 = 2\hat{A}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$

$= 2(\hat{A}_1 + \hat{A}_2)$

$B\hat{O}C = 2B\hat{A}C$

2.2.1 (a) $\hat{F}_1 = 2x$ ($\angle$ centre = 2$\angle$ at circum)

(b) $\hat{C} = x$ ($\angle$s in the same seg)

OR

$\hat{C} = x$ ($\angle$ centre = 2$\angle$ at circum)

2.2.2 $\hat{D}_3 = x$ ($\angle$s opp. equal sides)

$\hat{E}_3 = 2x$ (ext $\angle$ of $\triangle$)

$\therefore \hat{F}_1 = \hat{E}_3 = 2x$

$\therefore$ AFED is a cyclic quadrilateral (converse $\angle$s in the same seg)

2.2.3 $\hat{A}_2 + \hat{A}_3 + \hat{D}_1 + \hat{F}_1 = 180°$ (sum of $\angle$s in $\triangle$)

$\hat{A}_2 + \hat{A}_3 = \hat{D}_1$ ($\angle$s opp. = sides)

$\therefore \hat{A}_2 + \hat{A}_3 = 90° - x$

$\hat{E}_1 = \hat{A}_2 + \hat{A}_3$ (ext $\angle$ of cyclic quad.)

$= 90° - x$

$F\hat{K}E = 90°$ (line from centre bisects chord)

$\hat{F}_3 = x$ (sum of $\angle$s in $\triangle$)2.2.4 $B\hat{A}C = \hat{D}_3$ ($\angle$s in the same seg)

AE = BE (sides opp. equal $\angle$s)

$$\frac{\text{area } \triangle AEB}{\text{area } \triangle DEC} = \frac{\frac{1}{2}(BE)(AE) \cdot \sin A\hat{E}B}{\frac{1}{2}(EC)(ED) \cdot \text{in } D\hat{E}C}$$

$$6{,}25 = \frac{AE^2}{ED^2}$$

$$\therefore \frac{AE}{ED} = 2{,}5$$

Topic 8 Analytical geometry

Exam-type questions

1.1 $x = 3$

1.2 $m_{QP} = \tan 71{,}57°$

$= 3$

1.3 $y = mx + c$

$-2 = 3(-7) + c$

$y = 3x + 19$

OR

$y - y_1 = m(x - x_1)$

$y + 2 = 3(x + 7)$

1.4 R(3; 0)

$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-7 - 3)^2 + (-2 - 0)^2}$

$= \sqrt{104}$ or $2\sqrt{26}$

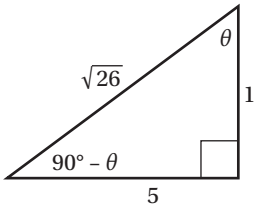
1.5 $\tan(90° - \theta) = m_{QR}$

$= \frac{0 - (-2)}{3 - (-7)}$

$= \frac{1}{5}$

1.6 $RN = \frac{1}{2} \cdot 2\sqrt{26} = \sqrt{26}$

$SR = 6$



Area $\triangle RSN = \frac{1}{2}SR \cdot RN \cdot \sin\theta$

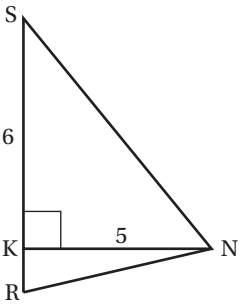
$= \frac{1}{2} \times 6 \times \sqrt{26} \times \frac{5}{\sqrt{26}}$

$= 15$ square units

OR

$SR = 6$

$\perp$ height $= 5$



$A = \frac{1}{2}SR \times \perp h$

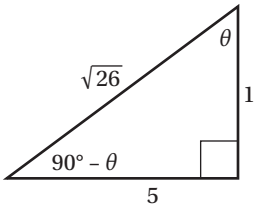
$= \frac{1}{2}(6)(5)$

$= 30$ square units

OR

$RN = \frac{1}{2} \cdot 2\sqrt{26} = \sqrt{26}$

$SR = 6$



Area $\triangle RSN = \frac{1}{2}SR \cdot RN \cdot \sin\theta$

$= \frac{1}{2}(6)\left(\frac{1}{2}QP\right) \cdot \sin\theta$

$= \frac{3}{2}(\sqrt{104}) \cdot \sin\theta$

$= 3 \cdot 2(\sqrt{104})\frac{5}{\sqrt{26}}$

$= 15$ square units

2.1 $OK = \sqrt{180}$ or $6\sqrt{5}$

2.2

$a^2 + b^2 = 180$

$b = \frac{1}{2}a$

$a^2 + \left(\frac{1}{2}a\right)^2 = 180$

$a^2 + \frac{1}{4}a^2 = 180$

$a^2 = 144 \quad \therefore a = -12$

$b = \frac{1}{2}(-12)$

K(−12; −6) (given)

OR

$a^2 + b^2 = 180$

$a = 2b$

$(2b)^2 + b^2 = 180$

$5b^2 = 180$

$b^2 = 36 \quad \therefore b = -6$

$a = 2(-6)$

K(−12; −6) (given)

2.3.1 $m_{OK} = \frac{1}{2}$ $(y = \frac{1}{2}x)$

$m_{PT} = -2$ (radius $\perp$ tangent)

$y = mx + c$

$-6 = -2(-12) + c$

$c = -30$

$y = -2x - 30$

OR $y - y_1 = m(x - x_1)$

$y - (-6) = -2(x - (-12))$

$c = -30$

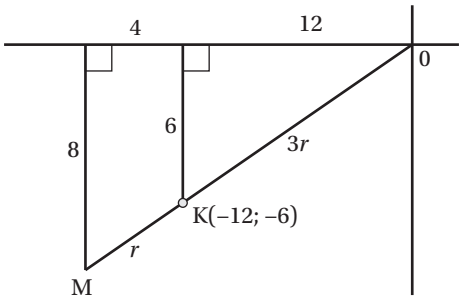
2.3.2 $3MK = OK$

$\Rightarrow OM = \frac{4}{3}OK$

$M = \frac{4}{3}(-12; -6)$

$\therefore M(-16; -8)$

OR



OR

$9MK^2 = OK^2 = 180$

$\therefore MK^2 = 20$

Let M(x; y), then $y = \frac{1}{2}x$:

$(x + 12)^2 + (y + 6)^2 = 20$

$(x + 12)^2 + \left(\frac{1}{2}x + 6\right)^2 = 20$

$4(x + 12)^2 + (x + 12)^2 = 80$

$(x + 12)^2 = 16$

$x + 12 \neq \pm 4$

$x = -16 \quad x \neq -8$ (since M is outside the large circle)

$y = -8$

M$(-16; -8)$

2.3.3 $(x - (-16))^2 + (y - (-8))^2 = \left(\frac{1}{3}\sqrt{180}\right)^2$

$(x + 16)^2 + (y + 8)^2 = 20$

2.4 $OK < r < OK + 2KM$

$\sqrt{180} < r < \sqrt{180} + \frac{2}{3}\sqrt{180}$

$6\sqrt{5} < r < 10\sqrt{5}$

2.5 $x^2 + 32x + (16)^2 + y^2 + 16y + (8)^2 = 256 + 64 - 240$

$(x + 16)^2 + (y + 8)^2 = 80$

New circle:

Centre: $(-16; -8)$ & $r = 4\sqrt{5}$

Original circle: M$(-16; -8)$ & $r = 2\sqrt{5}$

This circle will never cut the circle with centre M as they have the same centre (concentric circles) but unequal radii.

OR

$3MK = OK$

$9MK^2 = OK^2 = 180$

$\therefore MK^2 = 20$

Let M(x; y), then:

$(x + 12)^2 + (y + 6)^2 = 20$

$(x + 12)^2 + \left(\frac{1}{2}x + 6\right)^2 = 20$

$x^2 + 24x + 144 + \frac{1}{4}x^2 + 6x + 36 = 20$

$\frac{5}{4}x^2 + 30x + 160 = 0$

$x^2 + 24x + 128 = 0$

$(x + 16)(x + 8) = 0$

$x = -16 \quad x \neq -8$ (since M is outside the large circle)

$y = -8$

M$(-16; -8)$

$\therefore$ M$(-16; -8)$

Topic 9 Trigonometry

Exam-type questions

1.1 $\tan(180° + 41°) = \tan 41° = t$

1.2 $\sin 41° = \frac{t}{\sqrt{1 + t^2}}$

1.3 $\cos 82° = 1 - 2\sin^2 41°$

$= 1 - 2\left(\frac{t}{\sqrt{1 + t^2}}\right)^2$

$= \frac{1 - t^2}{1 + t^2}$

2.1 $\cos(A + B) = \cos A \cos B - \sin A \sin B$

2.2 $\cos(A + A) = \cos A \cos A - \sin A \sin A$

$= \cos^2 A - (1 - \cos^2 A)$

$= 2\cos^2 A - 1$

2.3 $\cos 3A = \cos(2A + A)$
$= \cos 2A \cos A - \sin 2A \sin A$
$= (2\cos^2 A - 1)\cos A - 2 \sin A \cos A \cdot \sin A$
$= 2\cos^3 A - \cos A - 2\sin^2 A \cdot \cos A$
$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$
$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$
$= 4\cos^3 A - 3\cos A$

2.4 $\cos^2 17° = k$
$\therefore \cos 17° = \sqrt{k}$
$\cos 51° = \cos^3(17°)$
$= 4\cos^3 17° - 3\cos 17°$
$= \sqrt{k}\,(4k - 3)$

3.1 $\cos A = -\frac{12}{13}$

3.2 $\cos(A + B) = \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(-\frac{5}{13}\right)\left(-\frac{3}{5}\right)$
$= \frac{48}{65} - \frac{15}{65}$
$= \frac{33}{65}$

4.1 $\text{LHS} = \frac{\sin x + 2\sin x \cos x}{\cos x + (2\cos^2 x - 1) + 1}$
$= \frac{\sin x(1 + 2\cos x)}{\cos x(1 + 2\cos x)}$
$= \frac{\sin x}{\cos x}$
$= \tan x$
$\therefore \text{LHS} = \text{RHS}$

4.2 $\text{LHS} = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$
$= \frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}$
$= \frac{1}{\sin A \cdot \cos A}$
$= \frac{1}{\frac{1}{2}(2\sin A \cdot \cos A)}$
$= \frac{2}{\sin 2A}$

5.1 $\sin(45° + x) \cdot \sin(45° - x) = (\sin 45° \cos x + \cos 45° \sin x)(\sin 45° \cos x - \cos 45° \sin x)$
$= (\sin^2 45° \cos^2 x - \cos^2 45° \sin^2 x)$
$= \left(\frac{1}{\sqrt{2}}\right)^2 \cos^2 x - \left(\frac{1}{\sqrt{2}}\right)^2 \sin^2 x$
$= \left(\frac{1}{\sqrt{2}}\right)^2 (\cos^2 x - \sin^2 x)$
$= \frac{1}{2}\cos 2x$

5.2 max. value of $\sin(45° + x)\sin(45° - x)$ = max. value of $\frac{1}{2}\cos 2x$
$= \frac{1}{2}(1)$
$= \frac{1}{2}$

6.1 $\text{LHS} = \cos(A - 30°) - \cos(A + 30°)$
$= (\cos A \cos 30° + \sin A \sin 30°) - (\cos A \cos 30° - \sin A \sin 30°)$
$= 2\sin A \sin 30°$
$= 2\left(\frac{1}{2}\right)\sin A$
$= \sin A$

6.2 $\sin A = 2\sin^2 A$
$\therefore 2\sin^2 A - \sin A = 0$
$\therefore \sin A(2\sin A - 1) = 0$
$\therefore \sin A = 0 \text{ or } \sin A = \frac{1}{2}$
$\therefore A = 0° + k.180°, k \in \mathbb{Z}$ or $A = 30° + k.360°$ or $150° + k.360°, k \in \mathbb{Z}$

7. $\sin(x + 60°) + 2\cos x = 0$

$\sin x \cos 60° + \cos x \sin 60° + 2\cos x = 0$

$\therefore \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 2\cos x = 0$

$\therefore \sin x + \sqrt{3}\cos x + 4\cos x = 0$

$\therefore \sin x = -(\sqrt{3} + 4)\cos x$

$\therefore \frac{\sin x}{\cos x} = -\sqrt{3} - 4$

$\tan x = -4 - \sqrt{3}$

8.1 $\cos(x + 30°) = 2\sin x$

$\therefore (\cos x \cos 30° - \sin x \sin 30°) = 2\sin x$

$\therefore \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - 2\sin x = 0$

$\therefore \sqrt{3}\cos x - 5\sin x = 0$

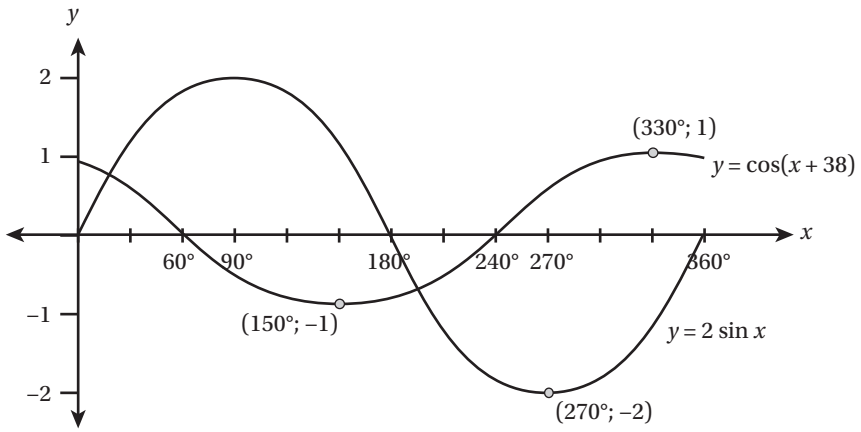
$\therefore \sqrt{3}\cos x = 5\sin x$

8.2 $\frac{\sqrt{3}}{5} = \frac{\sin x}{\cos x}$

$\therefore \tan x = \frac{\sqrt{3}}{5}$

$\therefore x = 19{,}1°$ or $199{,}1°$

8.3



8.4 $2\cos(x + 30°) \cdot \sin x < 0$ for $60° < x < 180°$ or $240° < x < 360°$

9. $\hat{D} = 43°$ (sum of angles in a triangle)

In $\triangle BCD$, $\frac{BC}{\sin 43°} = \frac{28}{\sin 96°}$

$\therefore BC = 19{,}2$ m

In $\triangle ABC$, $\cos B = \frac{BC}{AB}$

$\therefore AB = \frac{19{,}2}{\cos 30°}$

$= 22{,}2$ m

10.1 In $\triangle ABP$, $\hat{P} = 180° - (\beta + \theta)$

$\frac{AP}{\sin\theta} = \frac{x}{\sin(\beta + \theta)}$

$\therefore AP = \frac{x\sin\theta}{\sin(\beta + \theta)}$

In $\triangle APC$, $PC = AP\sin\alpha$

$\therefore PC = \frac{x \cdot \sin\alpha \cdot \sin\theta}{\sin(\beta + \theta)}$

10.2 $PC = \frac{7 \cdot \sin 42° \cdot \sin 68°}{\sin(75° + 68°)}$

$= 7{,}2$ m

Topic 10 Statistics

Exam-type questions

1.1.1 $\overline{x} = \frac{800}{10}$
$= \mathbf{80}$

1.1.2 $\sigma = 18{,}83$

1.2 (61,17; 98,83)
Days 1, 2, 8, 9 an 10 lie outside 1 standard mean $\therefore$ 5 days.

1.3.1 Skewed to the left or negatively skewed

1.3.2 A = 65, B = 99

1.4 New total = $95 \times 10 = 950$

2.1 Outlier: 100; 100)

2.2 $a = 94{,}50273 \ldots$
$b = 2{,}913729 \ldots$
$\hat{y} = 94{,}50 + 2{,}91x$

2.3 $\hat{y} = 2{,}91(240) + 94{,}50$
$= 792{,}90$
Value = R793 000
OR
$\hat{y} = 793{,}7978142$ (calculator)
Value = R794 000

2.4 $b = 2{,}913729 \ldots$
$\therefore$ R2 914 OR R2 910 (calculator)

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